PHYSICAL REVIEW E VOLUME 49, NUMBER 3 MARCH 1994

Stochastic resonance in a tunnel diode

R. N. Mantegna and B. Spagnolo
Dipartimento di Energetica ed Applicazioni di Fisica, Università di Palermo, viale delle Scienze, I-90128 Palermo, Italy
(Received 15 November 1993)

We study stochastic resonance in a fast bistable electronic system: a tunnel diode. We investigate the phenomenon in a higher frequency regime than that studied in previous experiments. Detailed measurements of the output signal are reported for two values of the frequency of the periodic signal: $f_s=1$ kHz and $f_s=10$ kHz. We observe, in one case ($f_s=1$ kHz), a nonmonotonic behavior characterized by a sharp dip in the output noise level measured at the frequency of the driving signal.

PACS number(s): 02.50.−r, 05.20.−y, 05.40.+j, 85.30.Mn

I. INTRODUCTION

During the last few years, there has been a growing interest in stochastic resonance [1]. The main effect of stochastic resonance (SR), first employed to explain geological glacial periods [2], is to enhance the response of a bistable system to a weak periodic driving signal by the injection of external noise. At the onset of the stochastic resonance, for appropriate noise intensity, the system performs quasiperiodic transitions between two coexisting attractors corresponding to the minima of a bistable potential.

The features of this phenomenon have been investigated in several theoretical analyses [2–8], with respect to different types of noise and potentials. In all the different approaches the Kramers escape-rate formula plays an essential role. On the other hand, the experimental investigations of stochastic resonance in physical systems are a limited number. Experiments of SR have been performed in a Schmitt trigger circuit [9], a bidirectional ring laser [10], analog simulations [11–13], an electron paramagnetic resonance system [14], and a magnetoelastic ribbon [15]. All these studies [3–15] focus the main attention on the behavior of the signal-to-noise ratio as a function of the input noise amplitude. The experimental results [9–15] have been limited to the range of low frequencies for the driving sinusoidal signal.

There is a lack of experimental investigation of bistable systems having very short switching times (of the order of nsec); due to this the highest frequency of the driving signal used up to now in experiments of stochastic resonance has been $f_s=3$ kHz [10].

In this Rapid Communication, we present an experimental investigation of the stochastic resonance observed in a fast bistable electronic device: the tunnel diode [16]. With a tunnel diode we realize a system which can perform fast transitions (roughly 40 nsec) between two stable states.

We report detailed measurements of the signal-to-noise ratio (SNR) observed in the spectral density of the output signal for two different values of the frequency of the driving sinusoidal signal: $f_s=1$ kHz, and $f_s=10$ kHz. Here, we are able to measure, separately, the output noise and signal levels observed at the frequency of the driving signal. By investigating the output noise levels we detect, in one case ($f_s=1$ kHz), a nonmonotonic behavior characterized by a well observable dip as a function of the external noise. A similar behavior has been theoretically predicted by McNamara and Wiesenfeld [4].

II. EXPERIMENTAL APPARATUS AND RESULTS

Our experimental setup consists of an electronic circuit, a noise source, and a detection line. The measurements are performed under software control through a personal computer (PC). The electronic circuit is very simple; it is the series of a tunnel diode with a resistor. By applying an appropriate constant voltage V to the series of resistor and tunnel diode we obtain a fast bistable electronic system. In our setup we choose $V=6.15$ V, $R=680\ \Omega$, and a germanium tunnel diode 1N3149A. This tunnel diode has nominal peak current $I_p=10.0$ mA, peak voltage $V_p=60$ mV, valley current $I_v=1.3$ mA, and valley voltage $V_v=350$ mV and the switching time is very short (close to 40 nsec).

The study of the SR phenomenon is performed by adding to the constant voltage a sinusoidal signal $V_s=V_s\cos(\omega t)$ and a Gaussian "white" noise $n(t)$. This is obtained by using a network of general purpose low-noise operational amplifiers (TL081).

The Gaussian noise is the output of a digital pseudorandom generator. The repetition interval of the noise cycle is 0.68 sec, a value which is higher or roughly equal to the time intervals used in our measurements. Our noise source provides noise with correlation time equal to 6.4 nsec. However, as in our experimental setup the noise is amplified, due to the finite value of the gain-bandwidth product of the amplifier the correlation time increases to 0.35 μsec. This correlation time implies a bandwidth of the noise of nearly 450 kHz.

By performing careful statistical analyses of our noise generator we verified that the probability density is very close to a Gaussian and the spectral density is almost flat (± 1.5 dB) at least up to 50 000 Hz (the value which is our experimental detection limit at the moment).

In our experiments we keep constant the amplitude of the periodic signal (we set it to the value $V_s=1$ V) and we vary the amplitude of the noise signal, i.e., we vary the root mean square of the noise $V_n=\sqrt{D}$. In the reported measurements, we vary V_n within the interval (500 mV–6.7 V).

The output of the system is the voltage across the tunnel diode. We detect the output signal by using a multifunction input output PC board. This board allows one to digitize up to 64 000 records with a maximal scan rate of 100 000 points per second.

The digitized time series consists of 8192 points typically. They are recorded with a scan rate of 1000 or 100 000 points per second and they are analyzed on-line by using a fast Fourier transform (FFT) routine. The output of the FFT routine is the spectral density of the output signal. We obtain more accurate spectral densities (Fig. 1) by averaging a number (usually 10) of different realizations of the process for a given setting of control parameters.

The measured spectral densities are analyzed off-line. By performing a fitting of an averaged spectral density with a Lorentzian curve, we are able to determine the value of the noise level observed by the frequency of the periodic driving signal with an accuracy of ± 0.5 dB. This high degree of accuracy is reached as we perform a fitting over 3000 digitized points of the averaged spectral density. In Fig. 1 we show the Lorentzian fitting of the spectral density. Moreover, from the spectral densities we determine the value of the power carried by the output signal at the frequency of the driving periodic signal.

Our first experimental investigation is devoted to verifying if our system performs transitions between the two stable states under the presence of external noise and in the absence of any periodic driving force. By increasing the input noise amplitude V_n, we detect output signals jumping randomly between the two stable states. The spectral density of these time series is always well fitted by a Lorentzian line shape:

$$S(f)=\frac{c}{(\gamma^2+f^2)} .$$

The values of γ experimentally determined as a function of the inverse of the noise variance D are shown in Fig. 2 in a logarithmic scale. In the same figure we report the straight line which is the best linear fitting of the experimental points. This line is given by the well-known Kramers formula [4]:

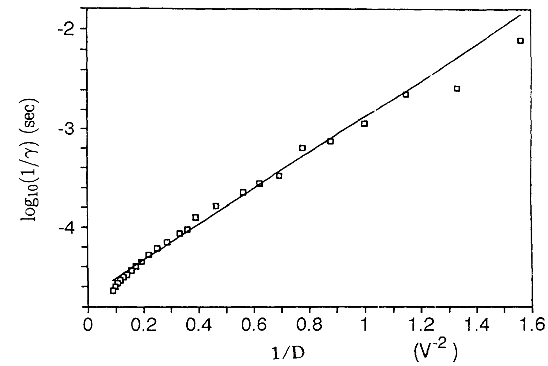

FIG. 2. Characteristic Kramers time $(1/\gamma)$ of our bistable system as a function of the inverse of the noise variance D.

$$\ln(1/\gamma)=\ln\left[\frac{\sqrt{2\pi}}{a}\right]+2\frac{U_0}{D} . \tag{1}$$

The best fitting is obtained by using $a=222\,000$ Hz and $U_0=2.6$ V; from the picture it is evident that the experimental points agree very well with the theoretical relation of Eq. (1). So the Kramers theory provides a good description of the time evolution of the system under the influence of an external noise. It is also worth noting that in the investigated range, i.e., up to values of $\gamma=40\,000$ Hz, we do not observe a deviation from the Kramers theory. This means that in our system, which may be the quickest used to investigate SR, the typical times are shorter than 10 μsec.

III. STOCHASTIC RESONANCE

We present studies of the stochastic resonance observed for two different values of the frequency of the driving periodic signal: $f_s=1$ kHz and $f_s=10$ kHz. We wish to point out that, in our setup, the study of the phenomenon at higher frequencies is not limited by the proper transition time of the tunnel diode but is limited by the finite recording time (10 μsec) of the multifunction input output board and by the finite bandwidth of our noise source. By improving the performance of the ex-

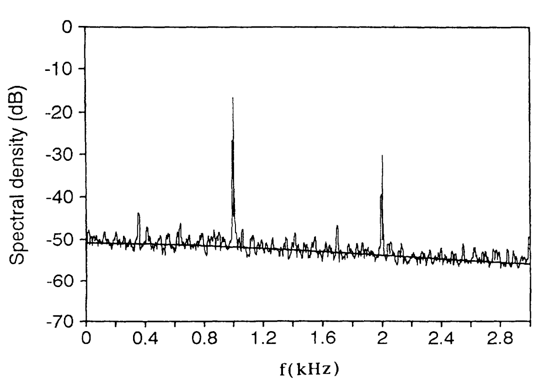

FIG. 1. Power spectral density of the output voltage. The main peak in the spectrum is at the frequency of the driving sinusoidal signal ($f_s=1$ kHz). The signal amplitude is $V_s=1$ V and the noise amplitude is $V_n=1.2$ V.

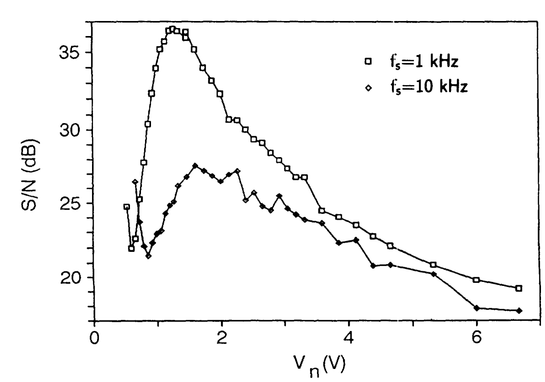

FIG. 3. Signal-to-noise ratio (SNR) as a function of the noise amplitude for two different values of the frequency of the driving signal ($f_s=1$ kHz, 10 kHz). In both cases $V_s=1$ V.

perimental setup the system could show stochastic resonance at much higher frequencies.

The results of our experimental investigations are summarized in Fig. 3, where we plot the signal-to-noise ratio as a function of the amplitude of the Gaussian noise for two values of the frequency of the driving signal. First we note that for small noise amplitude ($V_n \to 0$) the SNR grows rapidly in agreement with several theoretical models [4,6,8] and with physical intuition. In fact, in this region the system, driven by a small (or moderate) periodic signal and in the presence of a negligible amount of noise, performs oscillations of finite amplitude around one of the two stable points.

The overall behavior of the SNR versus the noise amplitude is peculiar to the stochastic resonance [1,4,9,10]: the SNR shows a rapid increase, a maximum, and then a slow decrease for higher values of the noise amplitude. At the higher frequency ($f_s = 10$ kHz) we observe a shift of the SNR peak and a flattening of the entire curve. This behavior, which cannot be ascribed to the breakdown of the adiabatic approximation (due to the short value of the switching time of the tunnel diode), is in qualitative agreement with theoretical results of McNamara and Wiesenfeld [4] and Jung and Hänggi [5]. However, a different explanation of this behavior is also possible. The shift and decrease of the SNR is observed at $f_s = 10$ kHz and the cutoff frequency of our noise is only $f_c \approx 450$ kHz so that it is also possible to ascribe these findings to the finite correlation time of the noise.

We are able to measure the signal and the noise output levels separately with an accuracy of ± 0.5 dB. In Fig. 4 we show the values of measurements performed to obtain the SNR data summarized in Fig. 3. In particular, Fig. 4(a) refers to the case $f_s = 1$ kHz whereas Fig. 4(b) refers to $f_s = 10$ kHz. From these figures it is evident that when the noise is maximum the signal is still increasing, then the noise falls off [Fig. 4(a)] or is almost flat [Fig. 4(b)], while the signal takes its maximum. It is this different behavior around the $D_{\max}$ (i.e., the value of input noise at which the SNR is maximum) which gives rise to the peak in the SNR (Fig. 3).

The experimental accuracy of our measurements enables us to detect in one case [$f_s = 1$ kHz, Fig. 4(a)] a nonmonotonic behavior of the noise output level with a characteristic noise dip. Moreover, both experimental results are in qualitative agreement with McNamara and Wiesenfeld's theory of stochastic resonance [4]. To rule out any doubt about the possibility of experimental bias we repeat the experiments with the same control parameters but with the sinusoidal driving signal off. The results of these investigations are compared with the previously shown results in Fig. 5. In Fig. 5(a) we compare the same data already presented in Fig. 4(a) (noise output levels) with the output noise levels measured at the same frequency ($f_s = 1$ kHz) when the driving signal is switched off. Figure 5(a) shows unambiguously that the observed nonmonotonic behavior is a real effect of the SR phenomenon. Figure 5(b) confirms that the nonmonotonic behavior is not observed for $f_s = 10$ kHz.

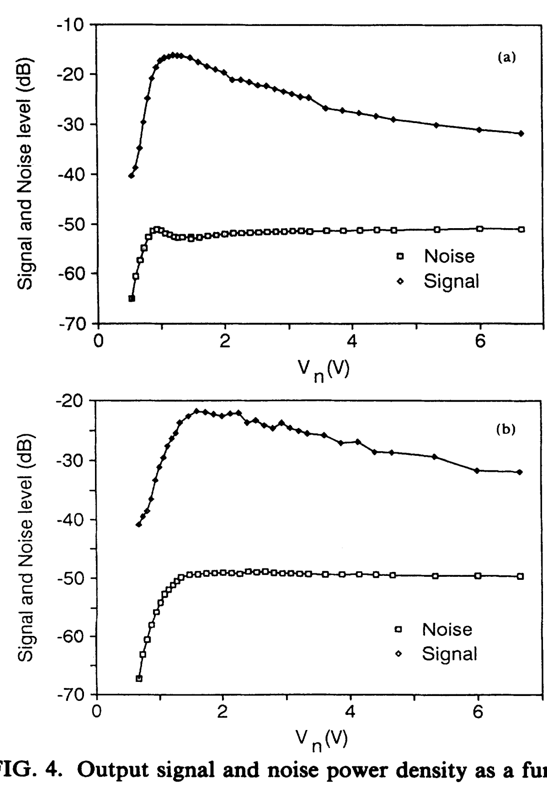

FIG. 4. Output signal and noise power density as a function of input noise amplitude V_n for two different values of the frequency of the driving signal: (a) $f_s = 1$ kHz; (b) $f_s = 10$ kHz.

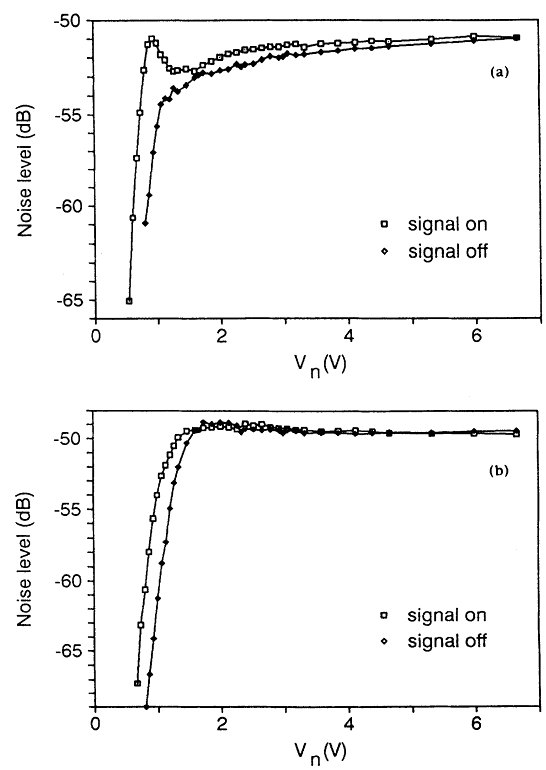

FIG. 5. Output noise power density vs input noise amplitude V_n in the presence of a sinusoidal driving signal (signal on, same data as in Fig. 4) and in the absence of a sinusoidal driving signal (signal off) for two frequency values: (a) $f_s = 1$ kHz; (b) $f_s = 10$ kHz.

In the measurements we have performed up to now with $f_s = 1$ kHz, we do not observe that the noise level detected in the presence of the sinusoidal signal is less than the noise level measured when the sinusoidal signal is off. This is in disagreement with the noise dip predicted by the theory of McNamara and Wiesenfeld [4], but it is physically acceptable. The discrepancy can probably be ascribed to the fact that the McNamara and Wiesenfeld theory does not take into account intrawell motion [12], whereas our experiments are performed on the output of our system without any filtering procedure.

In conclusion, we observe stochastic resonance in a bistable physical system characterized by a switching time very short and by a high degree of reliability. Our measurements are performed at relatively high values of the frequency of the driving signal and we are able to observe a nonmonotonic behavior of the output noise level with a peculiar noise dip.

The system is versatile enough to allow detailed experimental investigations of the SR phenomenon in a higher frequency regime and can be useful to check the results of different theoretical models.

ACKNOWLEDGMENTS

This research has been carried out using the experimental equipment of the Institute of Atomic Physics for Fusion, presently in the course of establishment according to an agreement between the CNR and the Sicilian Regional Government. The authors thankfully acknowledge the hospitality extended to them by Professor G. Ferrante, and wish to thank also Mr. M. Bonomo for his esteemed technical assistance. This work is supported in part by the MURST and the INFM.

[1] See, for example, Proceedings of the workshop Stochastic Resonance in Physics and Biology, special issue of J. Stat. Phys. **70**, 1 (1993); F. Moss, in *Some Problems of Statistical Physics*, edited by G. H. Weiss (SIAM, Philadelphia, 1992).

[2] R. Benzi, A. Sutera, and A. Vulpiani, J. Phys. A **14**, L453 (1981); C. Nicolis, Tellus **34**, 1 (1982); R. Benzi, G. Parisi, A. Sutera, and A. Vulpiani, *ibid.* **34**, 10 (1982).

[3] R. Fox, Phys. Rev. A **39**, 4148 (1989).

[4] B. McNamara and K. Wiesenfeld, Phys. Rev. A **39**, 4854 (1989).

[5] P. Jung and P. Hänggi, Europhys. Lett. **8**, 505 (1989); Phys. Rev. A **44**, 8032 (1991).

[6] P. Presilla, F. Marchesoni, and L. Gammaitoni, Phys. Rev. A **40**, 2105 (1989).

[7] G. Hu, G. Nicolis, and C. Nicolis, Phys. Rev. A **42**, 2030 (1990).

[8] G. Hu, H. Haken, and C. Z. Ning, Phys. Lett. A **172**, 21 (1992); Phys. Rev. E **47**, 2321 (1993).

[9] S. Fauve and F. Heslot, Phys. Lett. **97A**, 5 (1983).

[10] B. McNamara, K. Wiesenfeld, and R. Roy, Phys. Rev. Lett. **60**, 2626 (1988).

[11] L. Gammaitoni, F. Marchesoni, E. Menichella-Saetta, and S. Santucci, Phys. Rev. Lett. **62**, 349 (1989).

[12] T. Zhou and F. Moss, Phys. Rev. A **41**, 4255 (1990).

[13] M. I. Dykman, R. Mannella, P. V. E. Clintock, and N. G. Stocks, Pis'ma Zh. Eksp. Teor. Fiz. **52**, 780 (1990) [JETP Lett. **52**, 141 (1990)].

[14] L. Gammaitoni, M. Martinelli, L. Pardi, and S. Santucci, Phys. Rev. Lett. **67**, 1799 (1991).

[15] M. L. Spano, M. Wun-Fogle, and W. L. Ditto, Phys. Rev. A **46**, 5253 (1992).

[16] L. Esaki, Phys. Rev. **109**, 603 (1958).